The Accuracy of Marine Shadow-band Sun Photometer Measurements of
Aerosol Optical Thickness and Ångström Exponent

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ABSTRACT

An analytical uncertainty propagation model is used in conjunction with laboratory and field data to quantify the uncertainty in measurements of the direct-normal irradiance, aerosol optical thickness, and Ångström exponent made with a ship-mounted Fast-Rotating Shadow-band Radiometer (FRSR). Total uncertainties in FRSR measurements of aerosol optical thickness are found to be 0.02-0.03 at the 95% confidence level (two standard deviations). The “lever-arm” effect, a salient characteristic of the Langley technique in which uncertainties in aerosol optical thickness measurements are reduced as the solar zenith angle increases, is essentially offset by orientation uncertainty. Lack of a “lever-arm” effect precludes Langley calibration of FRSRs while at sea; they must be calibrated on land. Uncertainties in FRSR measurements of the two-wavelength Ångström exponent are shown to depend strongly on the aerosol optical thickness, with the maximum uncertainty of 0.6 associated with clean, maritime airmasses.
1 Introduction

Measurements of the transmissivity of aerosols in the visible and near-infrared wavelengths over the world’s oceans are needed to evaluate satellite estimates of aerosol radiation transfer properties used in climate and ocean color studies (King et al., 1999; Gordon and Wang, 1994). Over-water measurements are difficult and, until recently, only a few usable measurements existed. The number of over-ocean measurements of aerosol optical properties is expanding due to the development and implementation of new shipboard instruments: hand-held sun photometers (Porter et al., 2001; Deschamps et al., 2002) and marine Fast-Rotating Shadow-band Radiometers (FRSR; Reynolds et al., 2001). Hand-held sun photometers are collimated instruments that are pointed at the solar disk by an operator and measure the direct beam irradiance. In contrast, FRSRs and other shadow-band radiometers measure the direct-beam irradiance by decomposing the irradiance field into its direct and diffuse components (Guzzi, et al., 1985; Harrison et al., 1994a). Unlike hand-held sun photometers, FRSRs are designed for long-term, unattended deployments. There are currently 5-10 FRSR’s deployed on various ships across the world’s oceans at any given time.

Radiometry on a ship has the special problem of platform motion. A research vessel typically will have a periodic rocking motion with a period of 5–15° and a standard deviation of ±1–5° and a mean tilt of magnitude ±1–2°, which is related to weight distribution, wind forcing, and the directional wave field. The motion stability will change slowly over hours or days, and more suddenly when the ship changes direction abruptly. Hand-held sun photometers operated on ships rely on the human operator for orientation and stabilization, while the FRSR relies on careful monitoring of ship motion.
and compensation for it in the radiation measurements. There are several different methods of monitoring platform motion ranging from a gyroscope to fluid-based pitch and roll sensors. Cost and operating environment are important issues when selecting the motion monitoring apparatus. In addition to platform motion, the optical detector system used on an operational device like the FRSR is subject to the harsh environmental conditions encountered on the ship’s deck. This can cause drift in the detector circuitry, diffuser optical properties, or interference filters. Lastly, the ship’s effluent can cause problems through deposition on the diffuser or contamination of the atmospheric signal.

This paper addresses uncertainty in marine FRSR measurements of aerosol optical thickness, $\tau_{\lambda A}$. Subjects ranging from the impact of platform motion on measurement uncertainty through uncertainties in calibration are discussed. A lofty goal, on land as well as at sea, is to measure $\tau_{\lambda A}$ with an uncertainty of approximately 0.02 for solar zenith angles from $0^\circ$ to $70^\circ$ ($3 \geq m \geq 1$, where $m$ is the atmospheric air mass. $m \approx \sec \theta_e$). The basic issues that generate uncertainty are examined using an analytical uncertainty propagation model. This analysis provides a framework for interpretation of marine FRSR measurements by the atmospheric and oceanographic sciences communities.

2 Marine FRSR Uncertainty Analysis

The reduction in the intensity of solar radiation as it passes through the atmosphere is quantified by the optical thickness, $\tau_{\lambda}$, where $\lambda$ is the wavelength. Using Beer’s solution to Lambert’s Extinction Law, $I_{\lambda N} = I_{\lambda T} \exp(-m\tau_{\lambda})$, yields

$$\tau_{\lambda} = m^{-1} [\ln(I_{\lambda T}) - \ln(I_{\lambda N})]$$

where $\tau_{\lambda} = \tau_{\lambda A} + \tau_{\lambda R} + \tau_{\lambda O} + \tau_{\lambda C}$. In this expression, the subscripts $A, R, O,$ and $C$
indicate the contributions to the optical thickness made by aerosols, molecular scattering (Rayleigh), ozone absorption, and clouds, respectively. The variable \( m \) is referred to as the air-mass and is approximately equal to \( \sec \theta_e \) for solar zenith angles less than \( 70^\circ \) where \( \theta_e \) is the solar zenith angle relative to the earth’s coordinate system. \( I_{\lambda N} \) is the measured irradiance of the solar beam referenced to a plane that is normal to the solar beam (excluding all scattered diffuse light), and \( I_{\lambda T} \) is the top-of-the-atmosphere irradiance prior to interaction with atmosphere. With the exception of \( m \) (under typical conditions), each term in (1) is wavelength dependent, as indicated by the subscript \( \lambda \). A coefficient known as the Ångström exponent, \( \alpha \), describes the wavelength dependence of \( \tau_{\lambda A} \). The Ångström exponent (Ångström, 1961) can be computed in many ways and in this paper is defined as

\[
\alpha = \frac{\ln \left( \frac{\tau_2}{\tau_1} \right)}{\ln \left( \frac{\lambda_1}{\lambda_2} \right)} \tag{2}
\]

where the subscripts 1 and 2 refer to two different wavelengths, \( \lambda_1 \) and \( \lambda_2 \).

Much about marine FRSR uncertainty can be learned from a simple theoretical model based on uncertainty propagation (Meyer, 1992; Taylor, 1982). The standard deviation, \( \sigma_x \), is used in this paper to represent any uncertainty in the measurement, systematic or random, of \( x \). The variance equation for \( z = f(x, y) \) about point \( P \) is

\[
\sigma_z^2 \approx \left( \frac{df}{dx} \right)_p^2 \sigma_x^2 + \left( \frac{df}{dy} \right)_p^2 \sigma_y^2, \tag{3}
\]

where \( x \) and \( y \) are independent variables and all covariances are assumed to be zero. This is the formula employed in this paper to propagate uncertainty. In this paper, the terms in (3) are computed using the maximum values as determined analytically or from measurements, thereby yielding a “worst case” estimate of the uncertainty in the standard
deviation of the measured quantity. If a covariance existed in the variance equation, it would contribute in a manner that would compound this "worse case" estimate, but this scenario is unlikely in the problem at hand.

The first step is to derive the variance equations for $\tau_\lambda$ and $\alpha$ for a sun photometer. The irradiance at the top of the atmosphere can be written as $I_{\lambda T} = I_{\lambda 0} \left( \frac{r_0}{r} \right)^2$, where $I_{\lambda 0}$ is the mean irradiance for the mean earth-sun distance, $r_0$. The variable $r$ is the earth-sun distance at the time of the measurement. Substituting this into (1) and solving for $\tau_{\lambda A}$ yields

$$\tau_{\lambda A} = \frac{1}{m} \left[ \ln I_{\lambda 0} - \ln I_{\lambda N} - 2 \ln r_0 r \right] - \tau_{\lambda R} - \tau_{\lambda O} - \tau_{\lambda C}. \quad (4)$$

When the Langley method (Shaw, 1983; Harrison and Michalsky, 1994b, Schmid and Wehrli, 1995) is used, the optical thicknesses are assumed constant during a sunrise and sunset, and a plot of $\ln(I_{\lambda N})$ versus $m$ can be extrapolated to $m = 0$ so as to provide an estimate of $\ln(I_{\lambda T})$. It is possible to cast (4) in terms of voltage rather than irradiance, thereby eliminating the radiometer calibration as a source of uncertainty, although a case can be made for using irradiance for the purpose of gathering data that can be used to test radiative transfer codes for atmospheres containing aerosol, one of the applications of the FRSR. A second advantage of using irradiance rather than voltage is that the absolute calibration of the radiometer can be compared to previously measured values of top-of-the-atmosphere irradiance.

According to (3), the variance of (4) is

$$\sigma_{\tau_{\lambda A}}^2 = \left( \frac{1}{m} \right)^2 \left[ \left( \frac{\sigma_{I_{\lambda 0}}}{I_{\lambda 0}} \right)^2 + \left( \frac{\sigma_{I_{\lambda N}}}{I_{\lambda N}} \right)^2 \right] + \sigma_{\tau_{\lambda R}}^2 + \sigma_{\tau_{\lambda O}}^2 + \sigma_{\tau_{\lambda C}}^2, \quad (5)$$

which is nearly identical to equation 11 in Russell et al., 1979 and similar in structure.
to equation A15a derived in Russell et al. (1993), although the variables propagated are
different in the latter. It is assumed that $\sigma_m^2 \simeq 0$ since the geographic position of the
radiometer and the time of the observation are known to a high degree of accuracy.
Also, astronomical distances are well known and do not fluctuate on the same scale
as $\sigma_{\tau_\lambda}^2$, so $\sigma_{\tau_0/r}^2 \simeq 0$. As discussed in Appendix A, failure to correct for $\tau_{\lambda R}$ and
$\tau_{\lambda O}$ in certain wavelength bands will severely impact the absolute value of $\tau_{\lambda A}$, but $\tau_{\lambda R}$
and $\tau_{\lambda O}$ are known to reasonable accuracy and fluctuate slowly, so their contribution to
measurement uncertainty in a collection of measurements from a single day is negligible
(i.e. $\sigma_{\tau_{\lambda R}}^2 \simeq \sigma_{\tau_{\lambda O}}^2 \simeq 0$). Efforts are made to measure $\tau_{\lambda A}$ in cloud-free conditions, so
$\sigma_{\tau_{\lambda C}}^2 = 0$.

The corresponding variance equation for $\alpha$ from (2) is

$$\sigma_{\alpha}^2 = \left\{ \ln \left( \frac{\lambda_1}{\lambda_2} \right) \right\}^{-2} \left[ \left( \frac{\sigma_{\tau_{\lambda_1}}}{\tau_{\lambda_1}} \right)^2 + \left( \frac{\sigma_{\tau_{\lambda_2}}}{\tau_{\lambda_2}} \right)^2 \right]. \quad (6)$$

Equations (5) and (6) are used to evaluate the uncertainty in marine FRSR measure-
ments. Because $\sigma_{\alpha}^2$ is a function of $\sigma_{\tau_{\lambda A}}^2$, the task is reduced to determining $\sigma_{\tau_{\lambda A}}^2$, or more
fundamentally $\sigma_{I_{\lambda 0}}^2$ and $\sigma_{I_{\lambda N}}^2$. In the sections that follow, the magnitudes of $\sigma_{I_{\lambda N}}^2$ and
$\sigma_{I_{\lambda 0}}^2$ are examined; the foundation for these estimates is the transfer equation of a marine
FRSR, which is discussed in the following section.

a. The Transfer Equation of a Marine Shadow-band Radiometer

The marine FRSR described in this paper incorporates a seven-channel (one
broadband, six 10-nm narrowband) silicon-detector-based optical head and a semi-circular
occulting arm. The occulting arm circumscribes a complete rotation centered on the op-
tical head, thereby occulting a band of the sky, with a revisititation period of 6.5 seconds.
The signals from all seven optical channels are sampled rapidly so the shadow cast by
the shadow-band onto the detectors can be recognized and recorded. Reynolds et al.
(2001) describe the marine FRSR and develop all of the mathematics required to com-
pute $\tau_\lambda$. Two irradiance measurements must be determined from the high-speed-sampled
data stream during the shadow-band sweep over the upper hemisphere. These are the
edge irradiance, $I_{\lambda E}$, and the shadow irradiance, $I_{\lambda S}$. The “edge” irradiance is defined
as the most representative value of the measured irradiance when the shadow band is
adjacent to, but not occulting, the solar disk. Platform tilt measurements in the form of
pitch and roll are recorded twice each shadow-band revolution. Finally, mean values of
the platform’s latitude, longitude, and heading are measured.

The direct-normal irradiance for an FRSR is

$$I_{N,\lambda} = \frac{I_{\lambda E} - I_{\lambda S}}{\chi(\phi_r, \theta_r) \cos \theta_r}. \quad (7)$$

where the function $\chi(\phi_r, \theta_r)$ is a cosine correction factor for the optical head and $\phi_r$ and
$\theta_r$ are the solar azimuth and zenith relative to the instrument (Figure 1). The diffuser
material may have non-Lambertian scattering properties that combine with filter and
detector geometry to slightly alter the cosine response of the instrument; these effects
are quantified by $\chi(\phi_r, \theta_r)$. The cosine correction term is provided by the manufacturer
as a $\phi_r, \theta_r$ lookup table unique to each detector in the head. An important feature
in the design of the marine shadow-band radiometer is that $I_{\lambda E}$ and $I_{\lambda S}$ compensate
for the diffuse irradiance blocked by the shadow-band and are measured in a few tenths
of a second, so there is a negligible change in the $\phi_r, \theta_r$ pair during the measurement.
This compensation for the diffuse irradiance is not perfect if the diffuse field is highly
anisotropic, which occasionally occurs when the atmosphere is highly polluted or the
solar zenith angle is large. It is acceptable under most circumstances, though.

A linear calibration equation is used to convert from electronic analog-to-digital
converter output to measured irradiance, i.e. \( I_\lambda = g_\lambda v_\lambda + b_\lambda \), where \( v_\lambda \) is the analog-to-
digital converter output, \( g_\lambda \) is the calibration gain and \( b_\lambda \) is the offset. The calibration
equation is derived from factory calibrations of the optical head and from electronic cali-
brations, and the resulting calibration is unique to each of the seven channels in the head.

For a given FRSR, the electronic calibration changes by only tenths of a percent over a
year of operation while changes in the calibration of the optical head are considerable
and systematic, providing one of the major sources of uncertainty. The optical head
calibration issues will be discussed later. From this point on, the subscript \( \lambda \) refers to
one of the six narrow-band channels, and the wavelength-dependent irradiance, \( I_\lambda \) refers
to the mean irradiance weighted by the wavelength response of the channel, given by

\[
I_\lambda = \frac{\int I(\lambda) w(\lambda) d\lambda}{\int w(\lambda) d\lambda}.
\]  

(8)

The passband response \( w(\lambda) \) is provided by the manufacturer and integration is made
over the passband range.

Following (7), the actual direct-normal beam irradiance is

\[
I_{\lambda N} \simeq \frac{g_\lambda \Delta v_\lambda}{\chi(\theta_r) \cos \theta_r}.
\]  

(9)

where functional dependencies are indicated by parentheses and \( \Delta v_\lambda = v_{\lambda E} - v_{\lambda S} \). While
the functional dependence of \( \chi \) on \( \phi_r \) appears to be similar in magnitude to its dependence
on \( \theta_r \), (Figure 2) the ship’s angular motion would have to be tens of degrees to realize this
dependence. Since the ship’s angular excursions are only a few degrees, the functional
dependence of $\chi$ on $\phi_r$ is small and will be henceforth neglected. Additional evidence provided later in the paper shows that even if the functional dependence were to be of the same magnitude as the functional dependence on $\theta_r$, compared to other factors it would still be a negligible factor in the total uncertainty in the measurement of $\tau_{\lambda A}$.

The procedure for computing $\theta_r$ and $\phi_r$ begins with a calculation of solar azimuth, $\phi_e$, and solar zenith angle, $\theta_e$, in the earth’s coordinate system from the latitude, longitude, and time. These angles are used to compute the Cartesian coordinates of a solar beam vector of unit length in the earth’s coordinate system. The well-documented rotational coordinate transformation matrix, $T$, for three independent rotations, maps the radiometer-following coordinate system to the earth’s coordinate system. The components of the solar beam vector in the earth’s coordinate system are converted to the platform frame of reference by inverting $T$. The outputs of this conversion are the $x_r$, $y_r$, and $z_r$ components of a unit vector representing the position of the sun relative to the radiometer normal. The radiometer-relative solar zenith angle is computed using $\theta_r = \arccos(z_r)$.

The transfer equation given by (9) describes a marine FRSR in which the position of the radiometer is precisely monitored as the band occults the sun during each sweep. This equation represents the best possible equipment configuration, but not the most cost effective. The marine FRSR was designed to be a robust, reasonably-priced, autonomous instrument that could be integral to a global ship observing network. Instantaneous measurements of pitch, roll, and heading require sophisticated instrumentation, beyond the design goals, to separate true rotation from linear accelerations common in wave motion. A compromise is to use a damped-fluid, pitch-roll sensor and average all pitch
and roll measurements over a two-minute time period. Pitch and roll of the ship in a wave field is random, while mean tilt is systematic. The two-minute averaging time is sufficient to reduce the 5–10 secs wave-induced motions to an acceptable level, leaving the much slower mean tilts that result from ship list and wind/wave pressures. Averaging over two minutes eliminates the random element, leaving the mean tilt as a residual. As shown in Reynolds et al. (2001), this averaging provides an acceptable motion correction. Each pitch-roll sensor is calibrated at the factory. Temperature dependency is less than 0.5° over a 10–50° range. FRSRs are installed on large research vessels on which mean tilts are usually less than 2° and rocking seldom exceeds 5°. In these conditions, the two-minute averaged pitch and roll have a combined systematic and random uncertainty of approximately 0.5°.

The instantaneous direct-beam irradiance onto a horizontal surface is estimated by

\[
\langle I_{\lambda,H} \rangle \approx \langle I_{\lambda,E} \rangle - \langle I_{\lambda,S} \rangle = g_{\lambda} \langle \Delta v_{\lambda} \rangle
\]

where the angle brackets indicate an average over the two-minute interval. Following (7), the resulting equation for computing the normal-beam irradiance becomes

\[
\langle I_{N,\lambda} \rangle \approx \frac{g_{\lambda} \langle \Delta v_{\lambda} \rangle}{\chi_{r}(\phi_{r}, \theta_{r}) \langle \cos \theta_{r} \rangle}.
\]

The implications of averaging on the uncertainty will be discussed in later sections. The variance of \( \langle I_{N,\lambda} \rangle \) is

\[
\sigma_{\langle I_{N,\lambda} \rangle}^{2} \approx \left( \frac{g_{\lambda} \langle \Delta v_{\lambda} \rangle}{\chi \langle \cos \theta_{r} \rangle} \right)^{2} \left[ \left( \frac{\sigma_{g_{\lambda}}}{g_{\lambda}} \right)^{2} + \left( \frac{\sigma_{\langle \Delta v_{\lambda} \rangle}}{\langle \Delta v_{\lambda} \rangle} \right)^{2} + \left( \frac{\sigma_{\langle \cos \theta_{r} \rangle}}{\langle \cos \theta_{r} \rangle} \right)^{2} \right].
\]
In this expression, the cosine correction term \((\sigma_{\chi r}/\chi r)^2\) has been omitted because it is negligible compared to the other terms within the operating range of the FRSR \((\theta_e < 65^\circ)\). In fact, one reason that the operating range is limited to \(\theta_e < 65^\circ\) is to avoid the strongly non-linear portion of the cosine correction term. Omission of the cosine correction term can be justified by examining the magnitude of actual cosine correction terms, \(\chi_r(\phi_r, \theta_r)\), for a typical FRSR (Figure 2). The figure shows an exaggerated range for typical wave motion \((\sim 10^\circ)\) over a two-minute averaging period. Observations that fall within this two-minute period are averaged to produce an estimate of the mean tilt, as shown. This mean tilt is subject to \(0.5^\circ\) measurement uncertainty that occurs because the cosine correction term, \(\chi_r\), must be interpolated between laboratory measurements. This figure shows that \(\sigma_{\chi r} \approx 0.005\) and \(\chi_r \approx 0.98\), so \((\sigma_{\chi r}/\chi r)^2 \approx 2.6 \times 10^{-5}\) in the calculation of \(\sigma_{\langle I_{N,\lambda} \rangle}^2\). In contrast, the other terms in brackets are shown later to be of the order of \(10^{-4}\), an order of magnitude larger than \((\sigma_{\chi r}/\chi r)^2\), thereby justifying its omission. These magnitudes have been confirmed using a model of \(\chi_r\) formed by summing multiple Gaussian functions, using it to compute \((\sigma_{\chi r}/\chi r)^2\), and including \((\sigma_{\chi r}/\chi r)^2\) in (12). Results show that the uncertainty in the cosine correction term is not a significant contributor to measurement uncertainty under typical conditions. Recent comparisons have shown, however, that the cosine correction function itself does change significantly over time, perhaps due to changes in the optical properties of the diffuser material, and research to determine the magnitude of these changes and their impact on uncertainty in \(\langle I_{N,\lambda} \rangle\) are ongoing.

b. Gain Uncertainty
The terms $g_\lambda$ and $\sigma_{g,\lambda}^2$ in (12) are the actual value and uncertainty of the wavelength-dependent optical and electronic gain, $g_\lambda$. The gain uncertainty is a function of many factors, including the optical properties of the Spectralon® diffuser and interference filters, which can change with time when exposed to the elements. Compared to the changes in these optical properties, the gain drift due to the electronic circuitry behind the detectors is negligible. The detector of the FRSR is calibrated using a National Institute of Standards (NIST) bulb. These bulbs have a reported absolute accuracy of 0.6%, although more conservative estimates are made in the literature (Schmid and Wehrli, 1995). The manufacturer of the FRSR head quotes the gain uncertainty to be 2%, which is reinforced by FRSR comparison experiments performed as part of calibration exercises related to various FRSR deployments.

Experience has shown that the gain of some FRSR heads may drift by a considerable amount (>10%) during long deployments (several months), so it is imperative that the heads be frequently calibrated. Techniques have been developed to track the drift in the FRSR head gain by comparing the data from the FRSR’s broadband channel with a more stable broadband instrument. While this technique is sensitive to drift in the $\lambda$ response of the diffuser and interference filters and to water vapor contributions in broadband measurements, it can be used qualitatively to access whether the observed gain drift was gradual or instantaneous over the period of deployment. If the amount of the drift in the head gain is known from pre- and post-cruise calibrations, it can be accounted for so as to reduce gain uncertainty. The procedure used is to apply a linear correction factor bounded at the beginning and the end of the cruise by the calibrations. The appropriateness of this linear interpolation can be judged qualitatively by examining
the broadband comparisons. Although calculations are performed for higher gain uncertainties, the typical gain uncertainty is assumed to be 2%. A 2% gain uncertainty most often represents a freshly calibrated FRSR during its first few weeks of deployment.

c. Edge-Shadow Voltage Uncertainty

The edge-shadow uncertainty term quantifies uncertainty associated with making the measurement of the edge-shadow voltage difference. Because the FRSR must employ a band to occult the sun, some of the diffuse irradiance is inadvertently occulted. Therefore, an actual measurement of the diffuse field cannot be made, although, as discussed previously, the edge-shadow technique is used to estimate the amount of band-occulted diffuse irradiance allowing the diffuse field to be estimated. The procedure that is currently used for this estimate is to record the time series of the voltage for each individual 6.5° sweep of the occulting band through the sun during a two-minute averaging period (Figure 3a). Away from the sun, the band is occulting the global irradiance field, but as it sweeps across the solar disk it occults the direct beam irradiance (the shadow), along with a small part of the diffuse irradiance field. The voltage time series for a series of individual sweeps during the two-minute averaging period can result in simple, well-defined edge and shadow voltages (Figure 3a) or more complicated configurations (Figure 3b-c). Some of the physical factors that cause variations in the edge-shadow time series from one sweep to the next are revealed by combining (4) and (9). The resulting expression is

\[
\tilde{\Delta}v_\lambda = I_{\lambda 0\lambda} g^{-1}_\lambda (r_0/r)^2 \cos \theta e \exp \left( - \left[ \tau_{\lambda A} + \tau_{\lambda R} + \tau_{\lambda O} + \tau_{\lambda C} \right] \sec \theta e \right),
\]

(13)

where \(\tilde{\Delta}v_\lambda\) is the instantaneous voltage difference. This expression shows that instantaneous edge-shadow voltage \(\tilde{\Delta}v_\lambda\) should be impacted by platform motion through its
dependence on $\theta_r$. The other variables in (13) vary slowly and do not change from one sweep to the next. Hence, it is anticipated that a series of individual sweeps will differ from one another due to platform motion; if there is no motion, the individual sweeps should be almost identical to one another.

Strictly speaking, expression (13) does not provide a complete picture of the mechanisms that must be considered when operating an actual FRSR. The edge-shadow concept is relatively simple to implement when the scattering characteristics of the aerosol do not vary from the edge of the solar disk on one side of the sun to the edge of the solar disk on the other. It becomes more complicated if the aerosol has a pronounced directional phase function (Figure 3c) or if the sun is low in the sky. In the latter case, if the ship’s motion geometry is such that the band sweeps from top to bottom across the solar disk, there will be a large gradient in the geometric path length traversed by photons originating just above the upper and just below the lower edge of the solar disk. In these cases, the edge voltage may be quite different from one side of the sun to the other. In practice, we employ a spline interpolation across the shadow region to produce the best estimate.

Inspection of the voltage time series plots when the solar zenith angle is low (sun is high in the sky) and there is little rocking (Figure 3a) shows that there is little variation in the shadow voltages. This is expected because the shadow voltage is comprised of diffuse irradiance only, although a small portion of the diffuse irradiance is occulted by the imperfect shape of the band. The integrated diffuse irradiance is not directional. Conversely, there is considerable sweep-to-sweep variability in the edge voltages in the time series (Figure 3a) due to platform motion as suggested by (13). The motion impacts
the direct beam irradiance, which is the major contributor to the edge voltage.

The edge-shadow voltage uncertainty, \( \left( \frac{\sigma_{\Delta v_{\lambda}}}{\langle \Delta v_{\lambda} \rangle} \right)^2 \), quantifies how well the correct edge and shadow voltages can be estimated from the time series data collected during the two-minute averaging period. If the sweep time series is simple and has little sweep-to-sweep variability, the selection of the edge and shadow values is straightforward and there is little uncertainty (<1%). This usually occurs when there is little rocking, the solar zenith angle is low, and optical thickness values are less than 0.2. If the FRSR is stationary (on land), the uncertainty is due exclusively to natural variability in the atmospheric properties and uncertainties are 0.5% or less. The sweep-to-sweep variability in the edge voltage increases when the ship is subject to increased rocking (Figure 3c). Individual sweeps are averaged to eliminate the impact of instantaneous wave motion, thereby producing an average voltage time series consistent with the mean tilt angle of the ship during the two-minute averaging period. The standard deviation of the edge voltage time series essentially represents the uncertainty in the edge-shadow voltage measurement because the standard deviation of the shadow voltage is so much smaller. Typical values of this uncertainty when the ship is rocking normally are \( \sim 1-2\% \) in most circumstances.

The time series shape may become even more complex due to strong scattering by aerosols in close proximity to the direct beam vector, which may occur when the optical thickness is large (>0.5). In this case, the directional scattering phase function of the aerosols creates a large halo around the sun and may cause the edge voltages from one side of the sun to the other to be asymmetrical (Figure 3c). Under these circumstances, the selection of the edge voltage can become rather complicated, introducing additional
uncertainty, occasionally as much as 3%. The spline fit produces a better estimate of
the edge voltage (Figure 3c) and improves performance in cases when the optical thick-
ness is unusually large. This improved performance has been documented in extensive
comparisons with hand-held columnated sun photometers in highly polluted conditions.

In summary, the edge-shadow voltage uncertainty is determined by several factors,
including the rocking motion of the ship, the performance of the software that determines
the edge-shadow voltages, the solar zenith angle, and the optical thickness. We can
estimate uncertainty from the data; in clear conditions it is $\sim$1-2%, but in exceptional
cases it can be as much as 3%.

d. Orientation Uncertainty

Uncertainties in the exact specification of $\theta_r$ are due to uncertainties in the mea-
surements of heading ($\beta_h$), pitch ($\beta_p$), and roll ($\beta_r$) angles. It is possible to estimate the
impact of these uncertainties using

$$
\sigma_{\cos \theta_r}^2 \simeq \{ \cos[\theta_r(\beta_r + \Delta \beta_r, \beta_p)] - \cos[\theta_r(\beta_r - \Delta \beta_r, \beta_p)] \}^2
$$

(14)

where $\Delta \beta_r$ is the accuracy specification of the roll transducers as supplied by the man-
ufacturer and $\theta_r$ is computed using the procedure outlined above. The roll is used in
(14) because it is typically larger than the pitch, but the angle used in this calculation is
arbitrary; motion along either axis has the same effect on the uncertainties. The orien-
tation uncertainty is minimized at low solar zenith angles due to the shape of the cosine
function, as shown in the next section.
0.1 Uncertainty in the Direct-Normal Irradiance, \((\sigma_{I_{\lambda N}}/I_{\lambda N})^2\)

The percentage uncertainty in the direct-normal irradiance can be computed using (14), a gain uncertainty of 2%, and an edge-shadow voltage uncertainty of 1.5%. A range of calculations were performed for different wave conditions (Figure 4). The orientation term was computed for \(\theta_r\)'s of 0\(^o\), \(\pm 4^o\) and \(\pm 8^o\), which represent the full range of wave motion typically observed on ship carrying FRSR’s. Orientation uncertainty (Figure 4a) occupies an envelope bounded by the -8, and 8\(^o\) angles; note that the minimum at 8\(^o\) represents a situation when the roll angle is exactly the solar zenith angle, so the uncertainty in the radiometer’s position straddles the radiometer normal and (14) is zero. This envelope represents the orientation uncertainty for typical instantaneous wave motions. As noted, these individual wave motions are averaged over a two-minute period so as to determine the mean tilt, which is normally \(\pm 2^o\). The envelope of the orientation uncertainty for the mean tilt is considerably more narrow (Figure 4a).

When the solar zenith angle is between 40\(^o\) and 55\(^o\), the orientation uncertainty surpasses the gain uncertainty and, thereafter, dominates the uncertainty in the direct-normal irradiance measurement (Figure 4a). The 2% gain uncertainty is not a function of the solar zenith angle and produces a 3% uncertainty in \(I_{\lambda N}\) that is compounded at high zenith angles by the orientation uncertainty (Figure 4b). It is possible to derive a good estimate of the impact of averaging the typical wave motions to determine the mean tilt by averaging the uncertainty estimates for the \(\pm 8^o\) extremes (black line with black dots). Comparing this calculation with that representing no wave motion, but orientation uncertainty (solid blue line), shows that the averaging procedure nearly approximates the
uncertainty in $I_{\lambda N}$ in the “no motion” scenario ($0^\circ$ roll), as discussed in Reynolds et al. (2001). This suggests that averaging the typical wave motions over a two-minute period, assuming they are random, does not produce a significant averaging bias due to the cosine response of the radiometer. Degradation in the gain uncertainty to 4% from 2% (dashed blue line) essentially doubles the percent uncertainty in the $I_{\lambda N}$ measurement at solar zenith angles less than $30^\circ$ showing that gain drifts occurring during long deployments may significantly impact instrument accuracy.

In summary, averaging and using the mean tilt for the platform correction apparently results in a minor increase in the uncertainty of the direct-normal irradiance. Future FRSRs should be capable of instantaneous corrections for each sweep. Note that the 2-3% uncertainty in \( \sigma_{\langle I_{N,\lambda} \rangle}^2 / \langle I_{N,\lambda} \rangle^2 \) due to the gain uncertainty alone (Figure 4a; black line) is presumably the uncertainty in this measurement if the FRSR were mounted on land.

3 Uncertainty in the Aerosol Optical Thickness and Angstrom Exponents

To compute the uncertainty of the aerosol optical thickness, $\sigma_{\tau_{\lambda A}}^2$, it is necessary to specify the value of the direct normal irradiance, $I_{\lambda N}$, which can be obtained from (4) as

$$\langle I_{\lambda N} \rangle = I_{\lambda 0} r_0 r^2 \exp(-m[\tau_{\lambda A} + \tau_{\lambda R} + \tau_{\lambda O}]),$$  \hspace{1cm} (15)

where clear sky conditions are assumed ($\tau_C = 0$). The value of $I_{\lambda N}$ is dependent on the season (i.e. through $(r_0/r)^2$) and the aerosol optical thickness, $\tau_{\lambda A}$. It also depends on the Ångstrom exponent, $\alpha$, which describes the wavelength dependence of $\tau_{\lambda A}$. The values of $I_{\lambda 0}$ used here and in calculations performed below are taken from Thullier et
al. (2002). Although the seasonality correction factor is negligibly small, the date is assumed to be 21 December 2000 for the simulations shown in the rest of this paper. Neglecting molecular scattering and ozone components in accordance with Appendix A, (6) is written as

\[
\sigma^2_{\tau,\lambda} = \left( \frac{1}{m} \right)^2 \left[ \left( \frac{\sigma_{I_{\lambda 0}}}{I_{\lambda 0}} \right)^2 + \left( \frac{\sigma_{\langle I_{\lambda N} \rangle}}{\langle I_{\lambda N} \rangle} \right)^2 \right],
\]

(16)

which is used hereafter.

\[a. \text{ Uncertainty in the Extra-Terrestrial Irradiance, } (\sigma_{I_{\lambda 0}}/I_{\lambda 0})^2\]

An important application of the FRSR is to provide evaluation data for radiative transfer codes. These codes assume an extra-terrestrial irradiance spectrum at the top of the atmosphere and allow this incoming radiation stream to interact with atmospheric aerosols (or cloud particles) for the purpose of determining the vertical profile of heating in the atmosphere and the quantity of radiation that eventually reaches the surface. There are a number of published extra-terrestrial spectra, sometimes based on satellite radiation measurements. Therefore, it is imperative that the irradiance measurements made by the FRSR be constrained by the same extra-terrestrial irradiance spectrum used in radiation transfer models, or by a close approximation with known biases relative to widely used spectra.

A problem arises when the best published values for the extra-terrestrial spectral solar irradiance differ by a significant amount (Figure 5). Most of the disagreement in the published spectra occur in the blue-yellow (<600 nm) and in the near infrared (>800 nm) region. The data in Table 1 where obtained by convolving the spectral response functions for the bands in a typical FRSR head with the published solar spectra. The
variability in the estimates in Table 1 reflects a combination of instrument uncertainty
and variation in the solar output spectrum. The $I_{\lambda_0}$ values for 415 nm have a difference
of 4.6% over the range shown, for 500nm 2.3%, for 615nm 2.6%, for 670nm 1.9% and for
870nm 7.3%. The measurements of the extra-terrestrial solar spectrum made from space
(Colina et al., 1996; Thuillier et al., 1998; and Thuillier et al., 2002) are considered to
be the best. From these, the Thuillier et al. (2002) values are chosen as the reference
spectrum for the FRSR because the data were collected during four different missions,
thereby making this the largest and most comprehensive data set to date.

As previously mentioned, $I_{\lambda_0}$ for sun photometers is generally determined through
the Langley technique. As discussed below, however, the Langley technique cannot be
used to calibrate FRSRs at sea due to orientation uncertainty. Langley plots for the
FRSR must be performed on land. Langley calibrations of FRSRs on land at Mauna
Loa, Hawaii; Mt. Lemmon Arizona; and on the roof of Building 490D at the Brookhaven
National Laboratory, Upton, NY; generally provide values of $I_{\lambda_0}$ that agree within 1-
2% of the Thuillier et al. (2002) values. Accordingly, the $I_{\lambda_0}$’s used to process FRSR
data are derived by convolving the instrument bandpass with the Thuillier et al. (2002)
values, rather than through frequent Langely analyses, as would be the case on land. The
instrument gain change is estimated using pre- and post-cruise calibrations and adjusted
in the data in post-processing, in an attempt to maintain a calibration consistent with
the Thuillier et al. (2002) values. At sea, aerosol optical thickness measurements are also
compared with hand-held sunphotometer data (when it is available) to track drift, and
FRSRs are calibrated in a laboratory as frequently as possible. For the purpose of linking
FRSR irradiances to radiation codes, it is appropriate to assume on the basis of experience
and the values in Table 1, that the uncertainty in the absolute extra-terrestrial calibration for a typical FRSR is approximately 1.5%, assuming that the Thuillier et al. (2002) values are accurate. Because there is significant uncertainty in the solar reference spectra itself, this uncertainty could be significantly larger, particularly at 870 nm, whereupon later calculations show results for uncertainties of 1.5% and 3.0% for guidance.

b. Results

Calculations of $\sigma_{\tau_{\lambda A}}$ for several different situations were performed (Figure 6). The first experiment shows $\sigma_{\tau_{\lambda A}}$ for optimal FRSR operating conditions, when the uncertainty in the extra-terrestrial irradiance is 1.5% and the gain uncertainty is 2%. Calculations for 0, ±2, and ±8° were made, representing the envelopes of typical wave motion, mean tilt, and no motion. The envelope of $\sigma_{\tau_{\lambda A}}$ is quite narrow through solar zenith angles of 60°, but degrades rapidly at low solar zenith angles in the ±8° case (typical wave motion). Conversely, 2° uncertainty in the mean tilt produces a negligible effect on $\sigma_{\tau_{\lambda A}}$ as compared to the no motion case. Averaging the ±8° typical wave motion extremes provides an estimate of the impact of averaging the individual sweeps taken a 6.5 s intervals over two minutes (black dots); as anticipated, this calculation shows that the averaging approximates the “no-motion” case, reinforcing the notion that averaging the typical wave motions to compute the mean tilt should produce acceptable results. The mean tilt on ships on which the FRSR is currently deployed is less than 2° which suggests $\tau_{\lambda A}$ can be measured with an FRSR to an accuracy of 0.03 or better under most circumstances, assuming that the gain uncertainties are 2% and the absolute accuracy of $I_{\lambda 0}$ is known to 1.5%. 

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An interesting experiment is to calculate the uncertainty of a land-based FRSR whose orientation is perfectly known and gain uncertainty is 2% and $I_{\lambda 0}$ uncertainty is 1.5%. This can be accomplished by omitting the orientation uncertainty term in (12). This calculation (black line on Figure 6) shows that the land-based unit benefits from the “lever-arm” property of the Langley technique, which allows $\tau_{\lambda A}$ to be estimated with increasing accuracy as the solar zenith angle increases. When an FRSR is mounted on a ship, the advantages of the “lever-arm” are essentially offset by the increasing orientation uncertainty at high solar zenith angles; this produces a nearly constant uncertainty in $\tau_{\lambda A}$ ($\sigma_{\tau_{\lambda A}}$) as a function of the solar zenith angle. Moreover, this offsetting effect suggests that the Langley technique itself cannot be performed under most conditions at sea. Thus, FRSRs deployed on ships must be calibrated on land.

Two additional experiments are to degrade the $I_{\lambda 0}$ uncertainty to 3.0% (dashed black line on Figure 6), suggesting a poor knowledge of the extraterrestrial spectrum, and the gain uncertainty to 4% (dashed blue line), suggesting significant drift over time. Either of these conditions can add 0.01-0.02 to $\sigma_{\tau_{\lambda A}}$. An increase in gain uncertainty is a much more likely scenario for an FRSR exposed to the elements during long deployments. It should be reinforced that the absolute uncertainty in $I_{\lambda 0}$ is a function of inaccurate specification of the reference spectrum and the ability to perform an accurate Langley calibration, while gain uncertainty is the result of degrading hardware components, such as interference filters and the diffuser material.

Companion experiments to calculate the uncertainty in the two-wavelength Ångström exponent, $\alpha$, using (6) were performed (Figure 7). Conditions were varied from clean ($\tau_{\lambda A} = 0.05$, $\alpha = 0$) to polluted ($\tau_{\lambda A} = 0.2$, $\alpha = 1$) air masses. The experiments
demonstrate that the uncertainty in measuring $\alpha$ ($\sigma_\alpha$) is a strong function of $\tau_{\lambda A}$ and $\alpha$. As in previous experiments, the envelope of uncertainty for typical wave motion ($\pm 8^\circ$ shows that (red dashed lines) $\sigma_\alpha$ is not particularly sensitive to the wave motion until the solar zenith angle exceeds $60^\circ$. The land versus ocean experiment once again reflects the fact that the “lever-arm” effect is offset by uncertainty in the position of the FRSR on a moving ship, producing a $\sigma_\alpha$ that is nearly independent of the solar zenith angle. Experiments with degradation in the uncertainty of knowledge of $I_{\lambda 0}$ (black dashed line) and with gain (blue dashed line) again show sensitivity to gain uncertainty.

The most striking aspect of the Ångström exponent uncertainty is that it is so much greater in clean air masses ($\tau_{\lambda A} = 0.05$, $\alpha = 0$), where it is approximately 0.7 in contrast to the polluted case, which is an order of magnitude lower. The plot also shows that this situation exists for land-based units. On the surface, this characteristic appears to be advantageous, since most applications of the FRSR involve polluted air; the downside is that it seems that the FRSR is unable to accurately measure the wavelength-dependent characteristics of marine aerosol (sea-salt). While these aerosols contribute minimally to the local radiation budget, their spatial coverage over the world’s oceans suggests that they could be important in the global radiation budget.

4 Conclusions

The analysis above presents an opportunity for data users to evaluate the uncertainty of each measurement made with the FRSR when it is deployed on a moving ship. Uncertainty in $\tau_{\lambda A}$ and $\alpha$ can be computed (or estimated from this analysis) on the basis of the radiometer’s orientation with respect to the sun, information about the quality of
the radiometer gain calibration, and the integrity of determinations of $I_{\lambda_0}$. Once the uncertainties have been computed for each observation, they can be used as the filtering criteria for quality control or for hypothesis testing. In a typical experiment, it was found that the uncertainty in measurements of $\tau_{\lambda,A}$ is $\sim 0.03$ at large zenith angles and $\sim 0.02$ at small zenith angles. This uncertainty degrades to $\sim 0.04$ at small solar zenith angles as the gain uncertainty doubles. Land-based FRSRs are likely to perform better, particularly at large solar zenith angles, due to the advantage of the “lever-arm” effect implicit in the Langley technique, perhaps reaching accuracies on the order of 0.02 or better. At sea, orientation uncertainty offsets the “lever-arm” effect, causing the uncertainty in measurements of $\tau_{\lambda,A}$ and $\alpha$ to be relatively sun-angle-independent, compared to land deployments. This offset also precludes use of the Langley technique at sea, so marine FRSRs must be calibrated on land. The Ångström exponent is difficult to measure in clean, maritime airmasses, but this is more a function of the Langley technique itself than an artifact of the wave motion; similar difficulties would be encountered if the FRSR were mounted on a stabilized platform at sea or on land. It has been shown that the second derivative of $\alpha\$, which is $\alpha'$, is related to the aerosol size spectrum (O’Neill et al., 2001). For this application, it is clear that $\alpha'$ will have the same sensitivities as $\alpha$, thereby making it difficult to retrieve information about the aerosol size spectrum in clean, maritime airmasses.

The FRSR uncertainties at sea are on the order of 0.02 – 0.03. This is slightly higher than uncertainties generally reported for land-based sunphotometers, which are typically 0.01 – 0.02 (Schmidt et al., 1999). There are several reasons that the FRSR uncertainties are slightly larger than those for land-based units including: orientation un-
certainty, exposure to the elements, and inability to perform frequent Langley calibrations while at sea. Nonetheless, the FRSR uncertainties are acceptable for many applications and enable reasonably accurate, continuous estimates of $\tau_{\lambda,A}$ at sea.

One motivation for this work is the utility of marine shadow-band measurements of $\tau_{\lambda,A}$ and $\alpha$ to validate satellite estimates of same. To serve this function, an important question is: how accurate is the satellite retrieval likely to be and are the FRSR measurements more accurate? The accuracy and precision of satellite retrieval algorithms varies according to the configuration of the sensors and the methods used to estimate uncertainty are not uniform. Most uncertainty estimates are based on comparisons with surface-based sensors while others are based on model calculations. In general, the accuracy requirements for surface-based sensors are different depending on which satellite retrieval and application is under scrutiny.

A workshop was held in 1997 for the purpose of discussing passive remote sensing of tropospheric aerosol and atmospheric correction for the aerosol effect (Kaufman et al., 1997). Based on this workshop, it was generally agreed that the accuracy of current multichannel satellite retrievals is estimated to be $\Delta \tau_{\lambda,A} = 0.03 - 0.05$, where $\Delta \tau_{\lambda,A}$ is the accuracy of the retrieval. A later estimate of the accuracy of the Moderate Resolution Imaging Spectroradiometer (MODIS) retrieval suggested that $\Delta \tau_{\lambda,A} = 0.01 \pm 0.05 \tau_{\lambda,A}$ over the Atlantic Ocean off the southeast coast of the US, based on comparisons with land-based sun photometers. This uncertainty estimate is better than the uncertainty projections made in the workshop in 1997, although the authors caution that more trials are needed in different aerosol regimes. The accuracy of satellite measurements of the Ångström exponent, $\alpha$, are less certain, although comparisons with surface data suggest
the systematic biases of 30% are possible (King et al., 1999). There is a caveat to these
uncertainty estimates, however, since satellite aerosol retrievals are not attempted when
$\tau_{\lambda A} \leq 0.1$. When $\tau_{\lambda A} \leq 0.1$, sea-salt aerosol is assumed to be present and $\tau_{\lambda A}$ is specified
as 0.1 and $\alpha$ is set to zero.

Based on the results of this workshop, one requirement for satellite validation
measurements is that $\sigma_{\tau_{\lambda A}} \leq 0.03$ and $\sigma_{\alpha} \leq 0.3\alpha$ for many satellites. The results pre-

sented here suggest that a statistically significant number of FRSR measurements will
have accuracy in the 0.02 range, since $\sigma_{\tau_{\lambda A}} \approx 0.03$ under many conditions, so the instru-
ment should be useful as a source of satellite data set validation in many instances. The
instrument is designed to endure harsh conditions routinely encountered while operating
continuously and autonomously on ships at sea. While it may be unable to achieve the
accuracy of land based units in many circumstances, it does provide a relatively accurate
survey of aerosol optical conditions in open regions of the ocean, particularly if the region
contains polluted air advected from continental areas. The simulations above suggest
that the FRSR if carefully calibrated can meet these satellite validation requirements.

A second requirement is to understand the accuracy of the assumptions that are
made when $\tau_{\lambda A} \leq 0.1$. Clearly, the most challenging situation for both satellite and
FRSR is measuring $\alpha$ in clean air masses. Fortuitously, this type of air mass is of lessor
consequence for ocean color measurements and probably for climate forcing, though there
are no definitive measurements and calculations to support the latter. It is likely that
the $\alpha$ in these air masses is near zero, however, due to the mean size of sea-salt particles,
which is on the order of the size of small cloud droplets.

A main conclusion of this study is that the uncertainty in the exact position of the
FRSR relative to the Earth’s coordinate system essentially offsets the “lever-arm” effect in the Langley technique. To a large extent, this is because the pitch and roll transducers used in the current FRSR design are accurate to only 0.5°. New technology that has recently become available promises to significantly reduce this uncertainty and eliminate the need for averaging. Hence, future versions of the FRSR promise to be considerably more accurate. Another important conclusion is that frequent calibrations are necessary to properly quantify gain and $I_{\lambda 0}$ uncertainty. Nonetheless, the current version of the marine FRSR appears to be a cost effective means of collecting large amounts of satellite validation data over the world’s oceans.
Acknowledgements

This work was supported under contract by the National Aeronautics and Space Administration (NASA) Sensor Intercomparison and Merger for Biological and Interdisciplinary Ocean Studies (SIMBIOS) Program and the US Department of Energy’s Atmospheric Radiation Measurement Program. The author’s wish to acknowledge many people who, in addition to their own duties on various research vessels, have kindly offered their time to monitor our instruments. The author’s also wish to acknowledge Kirk Knoblespiesse and Dr. Christophe Pietras of the NASA Goddard Space Flight Center and Dr. Lee Harrison of the State University of New York for many useful discussions of various aspects of shadow-band radiometry. Finally, the author’s acknowledge Scott Smith and Ray Edwards for invaluable technical assistance in the design of the marine FRSR.
Appendix A: Estimates of $\sigma^2_{\tau_{\lambda R}}$ and $\sigma^2_{\tau_{\lambda O}}$

Uncertainty in the amount of molecular scattering ($\sigma^2_{\tau_{\lambda R}}$) and ozone absorption ($\sigma^2_{\tau_{\lambda O}}$) impact $\sigma^2_{\tau_{\lambda A}}$ and $\sigma^2_{\alpha}$. Fluctuations in molecular scattering and ozone absorption are functions of atmospheric conditions, and the variance that they contribute depends on the accuracy with which the atmospheric conditions can be specified. Molecular scattering is well understood, so $\tau_{\lambda R}$ can be computed from theory using (Penndorf, 1957)

$$\tau_{\lambda R} = \left( \frac{p}{p_0} \right) \left( a_1 \lambda^4 + a_2 \lambda^2 + a_3 + a_4 \lambda^{-2} \right)^{-1}.$$ \hspace{1cm} (17)

The variance equation for (8) is written as

$$\sigma^2_{\tau_{\lambda R}} = (p_0 \left[ a_1 \lambda^4 + a_2 \lambda^2 + a_3 + a_4 \lambda^{-2} \right])^{-1} \sigma^2_p$$ \hspace{1cm} (18)

where $(a_1, a_2, a_3, a_4) = (117.25, -1.32, 3.20 \times 10^{-4}, -7.68 \times 10^{-5})$, $p_0 = 1013.25$ hPa, and $\lambda$ is given in microns. Uncertainty in the molecular scattering coefficients is assumed to be negligible. A conservative estimate of $\sigma_p$ is 60 hPa, which represents the full range of observed atmospheric pressures (980 – 1040 hPa) and therefore serves as an upper bound. With $\sigma_p = 60$ hPa, $\sigma_{\tau_{\lambda R}}$ ranges from a maximum of $1.7 \times 10^{-10}$ at 415 nm to a minimum of $9.1 \times 10^{-12}$ at 862 nm, which shows that molecular scattering does not contribute significantly to $\sigma^2_{\tau_{\lambda A}}$. To summarize, despite a reasonable error in the specification of $p$, the impact on $\sigma^2_{\tau_{\lambda A}}$ is negligible and is heretofore neglected. It is noted that failure to subtract $\tau_{\lambda R}$ when computing the actual value of $\tau_{\lambda A}$ (as opposed to the uncertainty) will lead to a large error in the 400–500 nm wavelength bands.

Ozone absorption is also well understood, so $\sigma^2_{\tau_{\lambda O}}$ can be computed from theory if the amount of ozone in the atmospheric column is known. The total ozone in a column,
$\xi$, is typically measured in units of $10^{-3}$ cm of gas reduced to standard temperature and pressure (Dobson Units). Ozone absorption is computed by multiplying the total column ozone by an absorption coefficient, $k_\lambda$, determined from laboratory measurements. The impact of ozone absorption on the optical thickness is expressed as

$$\tau_{\lambda O} = \xi k_\lambda$$

and the corresponding variance from the ozone contribution to $\sigma_{\tau_{\lambda A}}^2$ is

$$\sigma_{\tau_{\lambda O}}^2 = k_\lambda^2 \sigma_{\xi}^2 + \xi^2 \sigma_{k_\lambda}^2.$$

Assuming that $k_\lambda$ is known to great accuracy, $\sigma_{k_\lambda}^2 \simeq 0$. The upper bound of the ozone contribution to $\sigma_{\tau_{\lambda A}}^2$ can be computed by assuming that $\sigma_{\xi} = 200$ Dobson units, which is the planetary range typically observed by the Total Ozone Mapping Spectrometer satellite. The ozone absorption coefficients for the wavelength bands of the shadow-band radiometer are given in Table A1 (M. Wang, personal communication). Calculations show that $\sigma_{\tau_{\lambda O}} \approx 0.02$ at $\lambda = 610$ nm and $\sigma_{\tau_{\lambda O}} \approx 0.01$ at $\lambda = 660$ nm, and $\sigma_{\tau_{\lambda O}} < 0.01$ otherwise. Assuming that $\sigma_{\xi}$ is far less than 200 Dobson units, these calculations show that $\sigma_{\tau_{\lambda O}}^2$ may not contribute significantly to $\sigma_{\tau_{\lambda A}}^2$, even in the vicinity of 600–700 nm. Failure to include the impact of $\tau_{\lambda O}$ when computing $\tau_{\lambda A}$ will have a detrimental impact on the accuracy of $\tau_{\lambda A}$, particularly in the vicinity of 600–700 nm.
References


Tables

Table 1: The different estimates of $I_\lambda$ from literature. Six different solar spectra are presented here. Each spectrum was convolved with the band-pass transfer function from a typical 10-nm filter. The mean irradiance (W m$^{-2}$) for the six estimates and the percent difference from the means are shown for each radiometer band. The current “best” estimate of $I_\lambda$ is Thuillier et al. (2002), which is significantly different than other spectra for important wavelengths.

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<th>B90</th>
<th>C96</th>
<th>T98</th>
<th>T02</th>
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N-L84-Neckel and Labs, 1984; W86-Wehril, 1986;
B90-Berk et al., 1990; C96-Colina et al., 1996;
T98-Thuillier et al., 1998; T02-Thuillier et al., 2002.
Table 2. Ozone absorption coefficients for FRSR center wavelengths

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Figure Captions

Figure 1. The geometry of a shadow-band radiometer operated on a moving platform.

Figure 2. Typical angle correction, $\chi(\theta_r, \phi_r)$, for an FRSR. The correction is measured using a light beam and rotating the head over $180^\circ$, through south-to-north and east-to-west planes, in $1^\circ$ steps. Differences between the two planes come from non-symmetry in the location of the silicone cells with respect to the foreoptic diffuser. Contamination of the diffuser will affect the correction terms. Indicated on the plot are envelopes of the typical wave motion and mean tilt uncertainty, which is derived from two-minute averages of the raw platform motion data. Also indicated is the mean tilt and the uncertainty in the mean tilt.

Figure 3. (a-c) Several raw voltage time series that were used in a two-minute average. Not all of the eighteen sweeps during the two-minute averaging pass initial filtering. The bold circles indicate the best voltage estimate for each time index and the circle at index twelve is the edge voltage using a spline fit of the average voltages (bold circles) along the two edges. The “x” marks the position of a simple linear interpolation, which was used in early versions of the FRSR processing software. The triangle marks the average value of the shadow voltage. The uncertainty is computed using the sum of the standard deviations of the two edge values observed just before and after the shadow and the standard deviation of the shadow voltage. This sum of standard deviations is divided by the edge-shadow voltage determined using the spline fit and converted to a percentage.

Figure 4. (a) The individual components of uncertainty in FRSR measurement of the direct-normal irradiance, $I_{\lambda N}$, as a function of the solar zenith angle, $\theta_e$. The uncertainty
in instrument gain (black dashed line) is 2% and the uncertainty in the edge-shadow voltage measurement is 1.5% (blue dashed line). The orientation uncertainty envelopes for typical wave motion (red-dashed lines; roll or pitch angles of $\pm 8^\circ$) and typical mean tilt (green-dashed lines; roll or pitch angles of $\pm 2^\circ$) are plotted; (b) As in (a), but for the combined effects of gain, edge-shadow, and orientation uncertainty. The black dots result from averaging the typical wave motion ($\pm 8^\circ$), which is a random variable; this exercise demonstrates that averaging these random waves essentially causes no bias due to the cosine response of the radiometer. The blue dashed line represents a doubling of the gain uncertainty to 4% (edge-shadow voltage uncertainty remains 1.5%). The solid black line shows the result when there is no orientation uncertainty (land), while the gain uncertainty and edge-shadow uncertainty are 2% and 1.5%, respectively.

Figure 5. A comparison of 6 different extraterrestrial solar spectral irradiances (W m$^{-2}$) as a function of wavelength. The lower illustrations are high resolution plots of the 400–450 nm (violet-blue) and 850–890 nm (near-infrared) regions of the spectrum.

Figure 6. The uncertainty in the FRSR measurement of aerosol optical thickness, $\tau_{\lambda A}$, as a function of solar zenith angle, $\theta_e$. Under typical circumstances, the uncertainty in instrument gain is 2%, the uncertainty in the edge-shadow voltage measurement is 1.5%, and the uncertainty in the extra-terrestrial irradiance is 1.5% (family of curves indicated on plot). The orientation uncertainty envelopes for typical wave motion (red-dashed lines; roll or pitch angles of $\pm 8^\circ$) and typical mean tilt (green-dashed lines; roll or pitch angles of $\pm 2^\circ$) are plotted. The black dots result from averaging the typical wave motion ($\pm 8^\circ$), which is a random variable; this exercise demonstrates that averaging these random waves essentially causes no bias due to the cosine response of the radiometer. The blue dashed
line represents a doubling of the gain uncertainty to 4% (edge-shadow voltage uncertainty remains 1.5%). The black dashed line represents a doubling of the uncertainty in the extra-terrestrial irradiance to 3%.

Figure 7. As in (6), but for Ångstrom exponent. (a) Plots of $\sigma_\alpha(\lambda_1, \lambda_2)$ versus $\theta_e$ for a clean maritime air mass ($\tau_{\lambda A} = 0.05$, $\alpha = 0$), $\lambda_1 = 860$ nm and $\lambda_2 = 500$ nm, and (b) for a polluted air mass with ($\tau_{\lambda A} = 0.2$, $\alpha = 1$)