TCRSR Band Angles

(Draft 02) October 1, 2008

1 Coordinate system

The origin of the coordinate system is at the top-center of the MFR head diffuser. The TCRSR is aligned so the axis of rotation is N-S and the motor is to the south. In the earth plane coordinate system the y-axis points to north, the x-axis points to east, and the z-axis points vertically upward. In the shadowband coordinate system the y-axis is identical to the earth system and points north. The x-axis runs from the origin through the center of the band.

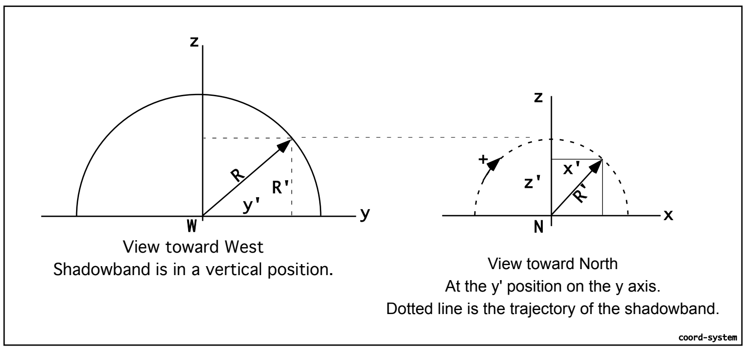

Figure 1: The coordinate system for the shadowband instrument. The left figure shows the x-z axis with the shadowband in a vertical position. The radius of the band is R and a vector from the origin to a point on the band in the vertical plane is defined by the angle η.

We define the following symbols:

ω The rotation rate of the band in deg/sample where sample is the sample count in the sweep.

η The position of a point along the band as an angle between a vector from the origin to the middle of the band. η has a range of $\pm 90°$ where $+90°$ points directly forward and -90° faces directly to the rear.

θ The band rotation angle measured from east to west. θ begins at 0° when the band is level to the east and increases to 180° when it is level to the west. An example might be the solar direct beam azimuth.

α The azimuth of a vector from the origin through any point on the centerline of the band. An example might be the solar direct beam elevation.

ϵ The elevation of a vector from the origin through any point on the centerline of the band.

α_s The azimuth of the solar beam in an earth coordinate system. α_s varies from 0° to north and increases in a clockwise direction.

ϵ_s The solar elevation angle. ϵ_s begins at 0° at the horizon and increases to 90° whan the sun is overhead.

For any position of the band, the $(x, y, z)_E$ coordinates of a point in the earth coordinate system is (see Figure 1)

$$x' = R\cos(\eta)\cos(\theta) \quad (1)$$
$$y' = R\sin(\eta) \quad (2)$$
$$z' = R\cos(\eta)\sin(\theta) \quad (3)$$

where R is the radius of the shadowband.

Given the coordinates of a point, the azimuth and elevation of the point relative to the origin is

$$\alpha = \tan^{-1}(x'/y') \quad (4)$$
$$\epsilon = \tan^{-1}\left(\frac{z'}{(x'^2 + y'^2)^{1/2}}\right) \quad (5)$$

We determine the the band rotation rate, ω, from the time difference from the start of the sweep to switch closure. The error in this number depends on the uncertainty in the start and stop angles but should be $< 2\%$.

$$\omega = \frac{(\theta_{limit} - \theta_{start})}{\Delta t}\,\tau \quad (6)$$

where θ_{limit} is the rotation angle at the closure of the limit switch, θ_{start} is the rest position of the band at the instant the sweep starts, and Δt is the time between the two θ positions.

2 Determining the Shadowband Rotation Angle

The exact position of the shadowband is crucial to interpreting the sweep data. However the current prototype system does not have an encoder by which we can know the exact position of the shadowband.

There are two indirect methods for computing the shadowband rotation angle: elapsed time and by use of an ephemeris.

2.1 Band position based on elapsed time

Given ω and θ_{start} one can compute the exact angle of the shadowband by

$$\theta_i = \theta_{start} + \omega\,(t_{delay} + i\,\delta\tau) \quad (7)$$

where $\delta\tau$ is the ADC sampling time in seconds/sample, i is the ith ADC sample from 0 to $(N-1)$, and t_{delay} is a small time allowed from the start of the sweep until the sampling commences.

There is some uncertainty in the elapsed time technique. The prototype TCRSR sometimes hangs up on the mechanical switches. The rest positions, θ_{start}, are not exact and can change during the operating period. Therefore for sweeps where there is a clearly defined shadow, the solar ephemeris can be used determine the band angle.

2.2 Band position based on the solar ephemeris

At the moment when the band is centered on the solar beam, $\alpha = \alpha_s$ and $\epsilon = \epsilon_s$. The solar angles are easily derived by an ephemeris. Therefore, we can determine the exact position of the band by solving the above equations for α_s and ϵ_s.

$$(\alpha_s, \epsilon_s) \Rightarrow (\theta_s, \eta_s) \tag{8}$$

The plan is to first use the computed solar angles to determine the exact position of the shadowband at the center of the shadow then compute the angle of the band using elapsed time around the minimum:

$$\theta_i = \theta_s + \omega\ (n_s - i) \tag{9}$$

where n_s is the sample count at the shadow point of the sweep, and i is the sample count.

The inversion is very simple. Given the azimuth and elevation angles, (α, ϵ),

$$A = \tan\alpha \tag{10}$$

$$E = \tan\epsilon \tag{11}$$

$$z = R\left[\frac{E^2}{1+E^2}\right]^{1/2} \tag{12}$$

$$y = \left[\frac{R^2 - z^2}{1+A^2}\right]^{1/2} \tag{13}$$

$$x = A\,y \tag{14}$$

Then the band angles, (η, θ) are computed by

$$\eta' = \sin^{-1}\ \frac{y}{R} \tag{15}$$

$$\theta' = \cos^{-1}\ \frac{x}{R\,\cos(\eta')} \tag{16}$$

3 Matlab equations

```
clear
% DEG TO RADIANS
D2R = pi/180;
R2D = 180/pi;

% TESTING -- DEFINE BAND ANGLES
R = 1;
rotang = [0:9:180]';
bandang = 45 * ones(length(rotang),1);

% compute (x,y,z) from band angles
x = R .* cos(D2R * rotang) .* cos(D2R * bandang);
y = R * sin(D2R * bandang);
z = R * sin(D2R * rotang) .* cos(D2R * bandang);
```

```
% compute azimuth and elevation from (x,y,z)
az = atan2(x,y) * R2D;
el = atan(z ./ sqrt(x.*x + y.*y)) * R2D;

% Use the azimuth and elevations to back-compute the (x,y,z)
E = tan(D2R * el);
A = tan(D2R * az);
zp = R * sqrt( (E .* E) ./ (1 + E .* E) );
yp = sqrt( (R .* R - zp .* zp) ./ ( 1 + A .* A ) );
ix = find( az > 90 & az < 270 & yp > 0);
if length(ix>0), yp(ix) = - yp(ix); end
xp = A .* yp;
ix = find( az > 180 & az < 360 & xp > 0 );
if length(ix) > 0, xp(ix) = - xp(ix); end

% compute band angles from the (x,y,z)
bandangp = asin( yp ./ R);
rotangp = acos( xp ./ (R .* cos (bandangp))) * R2D;
bandangp = bandangp * R2D;

return
```

3.1 Example of equations

Below are results from the above Matlab code. X, Y, Z are computed from the input azimuth and elevation and Xp, Yp, and Zp are computed from the inversion.

X	Y	Z	Xp	Yp	Zp
0.7071	0.7071	0	0.7071	0.7071	0
0.6984	0.7071	0.1106	0.6984	0.7071	0.1106
0.6725	0.7071	0.2185	0.6725	0.7071	0.2185
0.6300	0.7071	0.3210	0.6300	0.7071	0.3210
0.5721	0.7071	0.4156	0.5721	0.7071	0.4156
0.5000	0.7071	0.5000	0.5000	0.7071	0.5000
0.4156	0.7071	0.5721	0.4156	0.7071	0.5721
0.3210	0.7071	0.6300	0.3210	0.7071	0.6300
0.2185	0.7071	0.6725	0.2185	0.7071	0.6725
0.1106	0.7071	0.6984	0.1106	0.7071	0.6984
0.0000	0.7071	0.7071	0.0000	0.7071	0.7071
-0.1106	0.7071	0.6984	-0.1106	0.7071	0.6984
-0.2185	0.7071	0.6725	-0.2185	0.7071	0.6725
-0.3210	0.7071	0.6300	-0.3210	0.7071	0.6300
-0.4156	0.7071	0.5721	-0.4156	0.7071	0.5721
-0.5000	0.7071	0.5000	-0.5000	0.7071	0.5000
-0.5721	0.7071	0.4156	-0.5721	0.7071	0.4156
-0.6300	0.7071	0.3210	-0.6300	0.7071	0.3210
-0.6725	0.7071	0.2185	-0.6725	0.7071	0.2185
-0.6984	0.7071	0.1106	-0.6984	0.7071	0.1106
-0.7071	0.7071	0.0000	-0.7071	0.7071	0.0000

Below are the computed azimuth and elevation, (α, ϵ), for the input band angles, (η, θ).

Az	Elev
45.0000	0
44.6451	6.3508
43.5630	12.6214
41.7012	18.7246
38.9734	24.5588
35.2644	30.0000
30.4464	34.8941
24.4176	39.0528
17.1720	42.2602
8.8910	44.2989
0.0000	45.0000
-8.8910	44.2989
-17.1720	42.2602
-24.4176	39.0528
-30.4464	34.8941
-35.2644	30.0000
-38.9734	24.5588
-41.7012	18.7246
-43.5630	12.6214
-44.6451	6.3508
-45.0000	0.0000

The computed azimuth and elevations are applied to the inverse relationship to compute the band angles. Below are the original band angles, (η, θ) and the computed pair. They are identical.

band	rot	band	rot
45.0000	0	45.0000	0.0000
45.0000	9.0000	45.0000	9.0000
45.0000	18.0000	45.0000	18.0000
45.0000	27.0000	45.0000	27.0000
45.0000	36.0000	45.0000	36.0000
45.0000	45.0000	45.0000	45.0000
45.0000	54.0000	45.0000	54.0000
45.0000	63.0000	45.0000	63.0000
45.0000	72.0000	45.0000	72.0000
45.0000	81.0000	45.0000	81.0000
45.0000	90.0000	45.0000	90.0000
45.0000	99.0000	45.0000	99.0000
45.0000	108.0000	45.0000	108.0000
45.0000	117.0000	45.0000	117.0000
45.0000	126.0000	45.0000	126.0000
45.0000	135.0000	45.0000	135.0000
45.0000	144.0000	45.0000	144.0000
45.0000	153.0000	45.0000	153.0000
45.0000	162.0000	45.0000	162.0000
45.0000	171.0000	45.0000	171.0000
45.0000	180.0000	45.0000	180.0000

4 Apply to SGP data set

The subroutine `get_sweep_cal_v2.m` extracts a sweep from the SGP data set and computes the band angle based on the solar beam.

```
GET_CAL_SWEEP_V2
Get a designated sweep then use the calibration equations to compute
a calibrated and corrected output.

 typical call ::
 [dt,ang,sweep, glav, glsd,i1,i0,i2,toa] = ...
  get_cal_sweep_v2('/Users/rmr/data/tcrsr/c080704',1,2,50,datenum(2008,7,4,20,0,0));

input
 infolder = path to the folder with the hdr and swp files.
 band = 1 or 2 for 2 or 5 deg band
 chan = 0-7 for channel on MFR head
 nhalf = half width of output sweep.  Nsweep = 2*nhalf+1
 dts = datenum time.
output
 dt = datenum for this sweep
 angp = vector of angles for the sweep vector (centered on the solar beam)
 sweep = calibrated output
 glav = mean of global points on the ends of the full sweep.
 glsd = variance over the global points
 i1, i0, i2 = shoulder and min indexes
 toa = top of atmos flux from Thuillier 2002
```

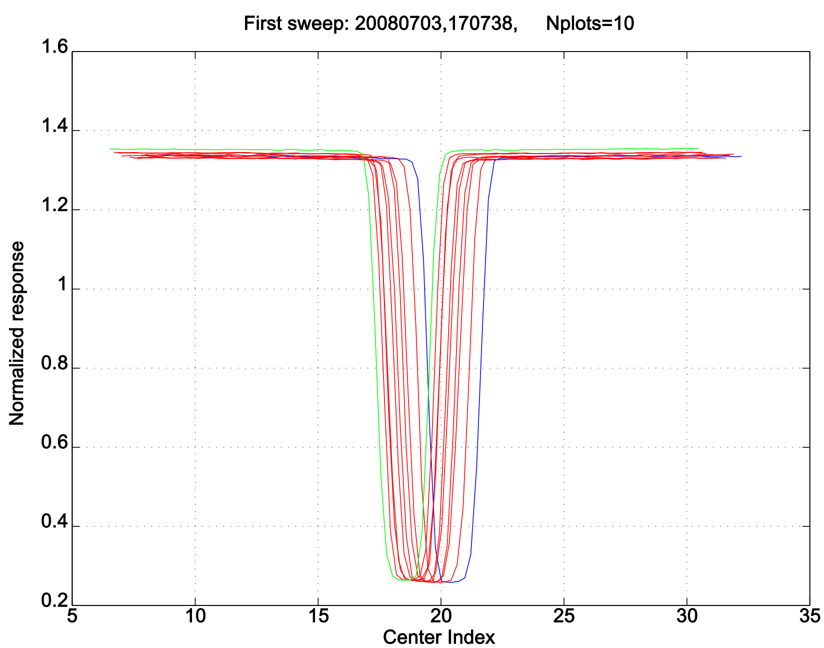

Figure 2: Ten sweeps with adjusted band angles. The first sweep is colored blue and the last sweep is green.